EXPERIMENTAL VERIFICATION OF DE BROGLIE'S HYPOTHESIS

ELECTRON DIFFRACTION

Purpose: With the electron diffraction tube, the particle and wave characteristics of electrons can be demonstrated and studied. In comparison with other experiments in the Quantum Physics of electrons, the method of electron diffraction on a crystal grid proves particularly advantageous because; the diffracted image can be made directly visible with the help of a fluorescent screen and only one simple compact test instrument to operate is required. The electron diffraction tube enables the de-Broglie's principle to be proved experimentally and with considerable accuracy, as a basis for a wave particle dualism (wavicle) applicable to electrons.

Theory: Max von Laue, in 1912, suggested that because of their regular arrangement of atoms, crystals might be used a diffraction gratings for X-rays. X-rays are electromagnetic radiation of about 1 A^o in wavelength, the same order of size as the inter atomic spacing in a typical crystal.

The theory of X-ray diffraction was developed by Sir William H.Bragg in 1913. Bragg showed that a plane of atoms in a crystal, called a Bragg plane, would reflect radiation in exactly the same manner that lights is reflected from a plane mirror, as shown in Figure 1.

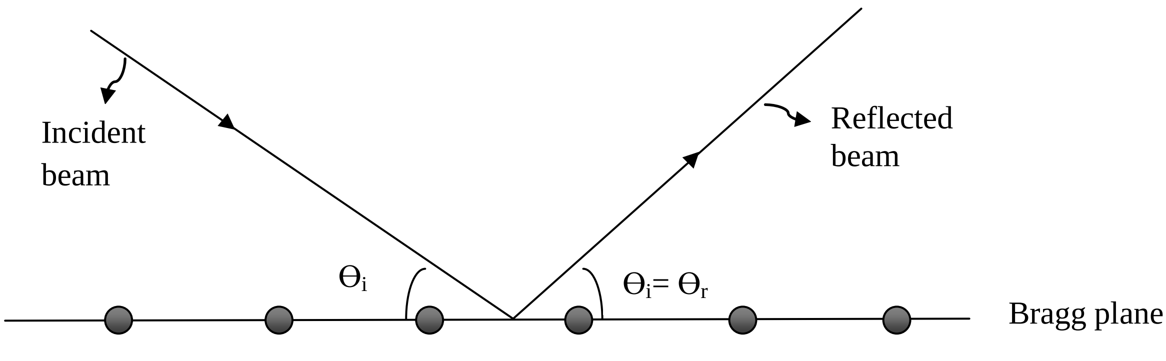


Figure 1

If one considers radiation that is reflected from successive parallel Bragg planes spaced a distance "d" apart, it is seen from Figure 2 that is possible for the beams reflected from each plane to interfere constructively to produce an enhanced overall reflected beam. The condition for constructive interference is that the path difference between the two rays $n\lambda=2d\sin\Theta$ be equal to as integral numbers of wavelengths, thereby giving Bragg's law as

$$n\lambda=2d\sin\Theta \qquad n=1,2,3,\ldots \qquad (1)$$

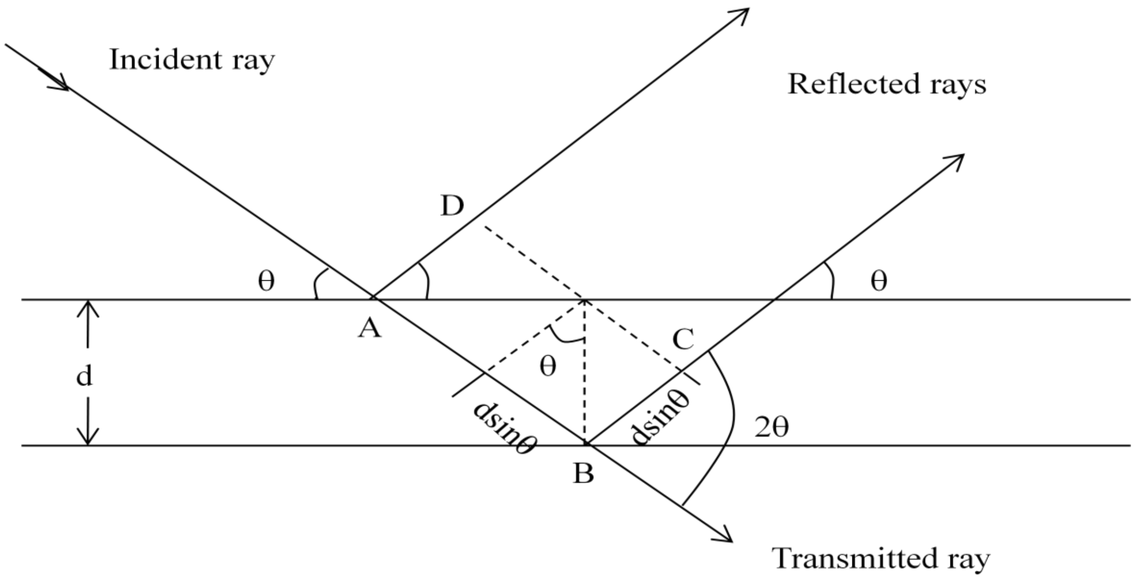


Figure 2

The first experiments to observe electron diffraction were performed by C.J. Davisson and L.H. Germer. Shortly after this experiment, G.P. Thomson, in 1927, studied the transmission of electrons through thin metal foils. If the electrons behaved like particles a blurred image would have resulted in the transmitted beam. Instead, Thomson found circular diffraction patterns, which can be explained only in terms of a wave picture, further confirming de-Broglie's hypothesis. Subsequently, thermal (low energy) neutron diffraction experiments were performed that further upheld the de-Broglie hypothesis.

Example: A narrow beam of 50 keV electrons passes through a thin silver polycrystalline foil. The interatomic spacing of silver crystals is 4.08 $\mathring{A}$. Calculate the radius of the first order diffraction pattern from the principal Bragg planes on a screen placed 40 cm behind the foil.

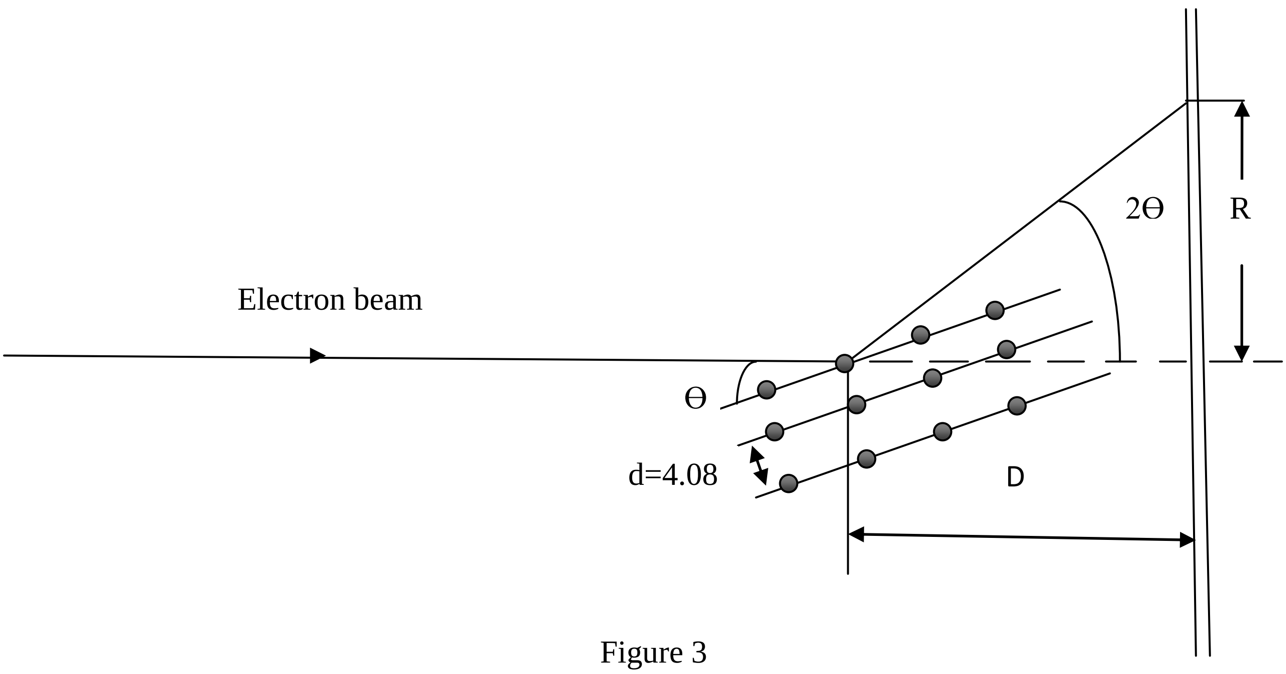


Figure 3

The de-Broglie's wavelength for the electron beam is

$$\lambda = \frac{hc}{pc} = \frac{hc}{\sqrt{E^2 - E_0^2}} = \frac{hc}{\sqrt{(K + E_0)^2 - E_0^2}} \qquad (2)$$

$$= \frac{12.4\times10^3 eV\ A^o}{\sqrt{(60\times10^3 eV\ +\ 511\times10^3\ eV)^2 - (511\times10^3\ eV)^2}} = 0.0487 A^o$$

For first order Bragg reflection;

$$\sin\theta = \frac{\lambda}{2d} = \frac{0.0487A^o}{2(4.08A^o)} \qquad (3)$$

From which sinΘ =0.342 from figure 3 the radius of the first order diffraction pattern is given by

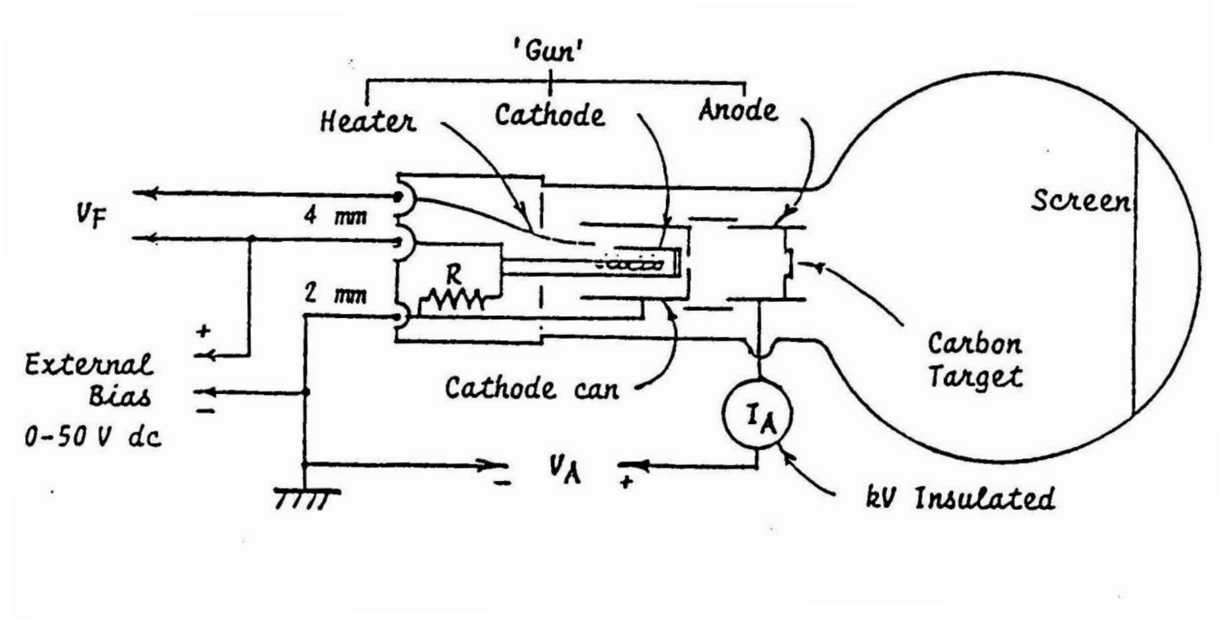
$$R = D\tan2\theta = (40cm)\tan 0.648 = 0.478\,cm$$

Procedure: A similar calculation can be made using Carbon and assuming that its atomic system is cubic, 12 gms of Carbon contain 6 x10^{23} atoms (Avogadro's number), the density of Carbon is about 2 gms per cm^3, 1 cm^3 contains 10^{23} atoms so that adjacent Carbon atoms will be $3\sqrt{10}$ or a little over 0.2nm apart. It is thus reasonable to expect that Carbon should provide a grating of suitable spacing for a diffraction experiment. Because a calculation using de-Broglie's equation shows that electrons accelerated true a potential difference 4 kV has a wavelength of about 0.002 nm.

Connect the electron diffraction tube in to the circuit shown in Fig. 4 switch on the heater supply and wait 1 minute for the cathode to heat stabilize. Adjust the E.H.T setting to 4.0 kV.

Specification:

Filament Voltage (V_F) …………………………………6.3 V ac/dc (8.0 V max.)
Anode Voltage (V_A)……………………………………...2500 - 5000 V dc
Anode Current (I_A)……………………………………… 0.15 mA at 4000 V (0.20 mA max.)



Note that connect the E.H.T negative to the 2mm socket only

Figure 4.

Two prominent rings about a central spot are observed, the radius of the inner ring being in fair agreement with the calculated value of 14 mm. Variation of the anode voltage causes a change in diameter, a decrease in voltage resulting in an increase in diameter. This is in accord with de-Broglie's suggestion that wavelength increases with decrease in momentum. Evidence of the particulate nature of the electron has been previously obtained and so this demonstration, which so closely resembles the optical one, reveals the dual nature of the electron.

The de-Broglie wavelength of a material particle is

$$\lambda = \frac{h}{mv} \tag{4}$$

Where "h" is Planck's constant. The velocity, "v" can be obtained from the classical expression

$$eV_a = \frac{1}{2}mv^2 \tag{5}$$

And substituted into the de-Broglie relation, obtaining

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2emV_a}} = 1.23 V_a^{-1/2} nm \qquad (6)$$

The condition for first order (m=1) diffraction for small angles is

$$\lambda = d\theta \qquad (7)$$

Where the small angle   can be calculated from the geometrical relationship of Figure 5

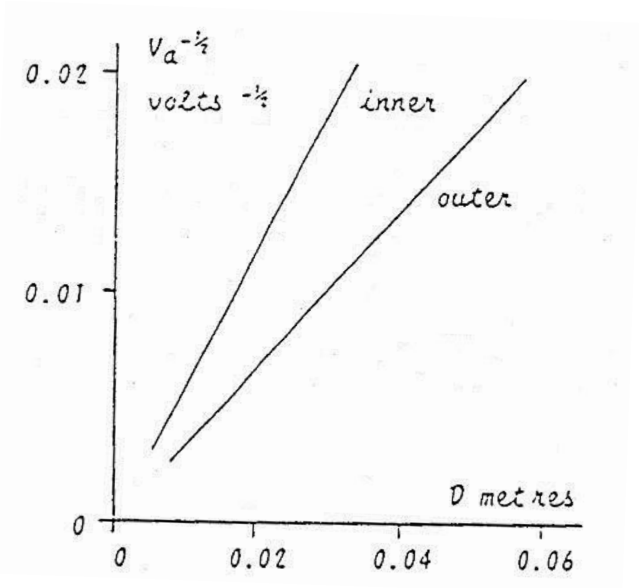
$$\theta = \frac{D/2}{L} \qquad (8)$$

And so from Eq.6

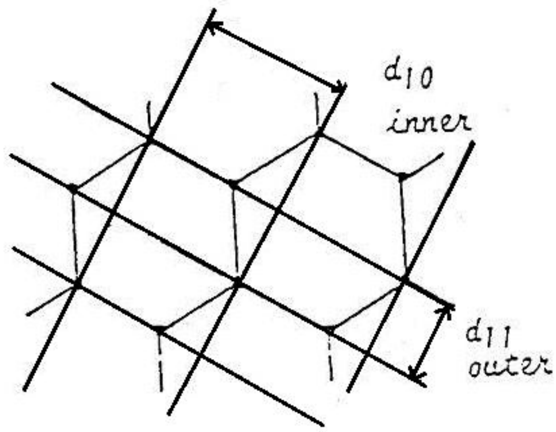
$$D\frac{d}{2L} = 1.23 V_a^{-1/2} nm \qquad (9)$$

D and   are the only variables; tabulate D for different anode voltages   and plot the graph D proportional to   .

(kV)	( )	D (meters)	
		Inner	Outer
2.5	0.0200		
3	0.0183		
3.5	0.0169		
4	0.0158		
4.5	0.0149		
5.0	0.0141		



Measure the pathlength from the carbon target at the gun exit aperture to the luminescent screen L m, as accurately as possible using back-reflection technique ($0.140 \pm 0.003m$).

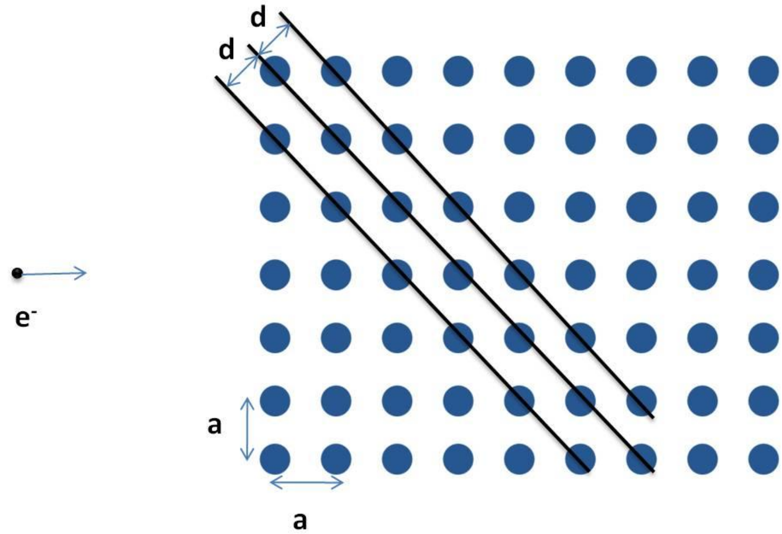


Rearrange the Eq.9 and evaluate the interatomic spacing d using the gradients of the graphs of the outer and inner circles, compare with the established figures of $d_{11}(0.123)$ and $d_{10}(0.213)nm$.

These result verify the theory and substantive the de Broglie hypothesis, note the arrangement of the spacing is $\sqrt{3}:1$ which suggests that the arrangement of the Carbon is most likely to be hexagonal rather than the assumed cubic.

QUESTIONS

1. A 0.083 eV neutron beam scatters from an unknown sample and Bragg reflection peak is observed centered at 22^o. What is the Bragg plane spacing?
2. A crystalline material has a set of Bragg planes separated by 1.1 A^o. For 2 eV neutrons, what is the highest order Bragg reflections?
3. Rock salt forms a cubic lattice with the sides of each plane having a length a=5.68 A^o. An electron beam is incident on a rock salt as shown in the figure below:
    a. What is the spacing between the lattice planes indicated by dashed lines?
    b. Through the smallest potential should the electrons be accelerated if they tend to be reflected strongly off the planes indicated by the dashed lines?



REFERENCES

1. The manual on the electron diffraction tube prepared by Teltron Limited.
2. R.Gautream and W.Savin, Modern Physics (Chapter 16). McGraw-Hill (1978)

PRELAB QUESTIONS

1. Prove that the formulation for the Bragg law is $2\, d \sin\varphi = n\, \lambda$
2. What is the aim of the electron diffraction experiment? What is the De Broglie hypothesis? In which part of the experiment we use de Broglie hypothesis and in which part we use Bragg law to explain the experiment. What is the relationship between wavelength and applied voltage?
3. Is this experiment valid only for electron or is it valid for different particles too? For example can we use proton instead of electron in this experiment, in that case what do we observe in the screen? Please explain.